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DIFFUSION FROM A LINE SOURCE IN A NEUTRAL OR STABLY STRATIFIED ATMOSPHERIC SURFACE LAYER

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Abstract—Diffusion of pollutants downstream of a crosswind line source in a turbulent boundary layer is investigated. A modified Van Driest model of turbulence is used to represent the turbulent diffusion. Effect of thermal stratification on turbulent diffusion is represented by a flux Richardson number. Numerical solutions of the governing boundary-layer partial differential equations are obtained using an implicit finite difference method. Results are presented and compared with experimental data and show good agreement. Both constant and variable turbulent Schmidt numbers are considered.

NOMENCLATURE

A^*, Van Driest constant;
c_p, specific heat at constant pressure;
g, gravitational acceleration;
G, source strength;
K, von Karman's constant;
k, thermal conductivity;
L, stability length scale;
l, Prandtl's mixing length;
$\dot{m}''$, mass flux;
m_1, concentration of species;
Pr, Prandtl number;
$\dot{q}''$, wall heat flux;
Re_x, Reynolds number, $= u_e x/\nu$;
Ri_f, flux Richardson number;
u, v, velocity components;
x, coordinate in streamwise direction;
x_s, distance downstream of the source;
y, coordinate perpendicular to wall;
y^+, dimensionless distance, $= y\sqrt{(\tau_w/\rho)}/\nu$;
y_l, characteristic thickness of the layer.

Greek symbols

α, thermal diffusivity;
δ, boundary layer thickness;
ε_D, turbulent eddy diffusivity for mass;
ε_H, turbulent eddy diffusivity for heat;
ε_M, turbulent eddy diffusivity for momentum;
θ, temperature;
λ, empirical constant, and y at which $m_1 = 1/2 m_{1,\max}$
μ, dynamic viscosity;
ν, kinematic viscosity, $= \mu/\rho$;
ω, cross stream coordinate;
ρ, density;
τ, shear stress;
Φ, generalized dependent variable;
ϕ, dimensionless wind shear;
Ψ, stream function.

Subscripts

e, freestream, outer edge of boundary layer;
eff, effective;
m, mean;
t, turbulent;
w, wall;
1, species 1.

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INTRODUCTION

DURING the past decade, conservation of the atmosphere surface layer (150–300 ft) became a major concern as a result of worsening air pollution due to industry and motor vehicles. In order to lessen harmful effects on people, animals and plants, it is essential to control and reduce air pollution to a minimum level. Quantitative predictions of pollutant concentrations and diffusion, both laminar and turbulent, in the atmosphere are important in establishing relationships between sources and receptors.

A number of experimental and theoretical works on atmospheric diffusion problems are available in the literature. Experiments include both field as well as laboratory experiments. An excellent summary of field experiments can be found in [1]. In laboratory experiments, the surface layer has been modeled in wind tunnel experiments, for example [2–5], to mention a few. Most of the theoretical work makes use of empirical correlations and diffusion models [1, 6].

Diffusion in the surface layer is controlled by wind driven turbulent mixing as well as the degree of thermal stratification. For example, under inversion conditions (stable stratification) the potential temperature increases with elevation; this leads to the suppression of the vertical turbulent mixing. In order to adequately describe diffusion in the atmosphere, turbulent diffusion as well as thermal stratification effects must be included in a given model. Many phenomenological theories are available in the literature to describe turbulent diffusion. For example, Bradshaw *et al.* [7], Harlow and Nakayama [8], Nee and Kovasznay [9] and many others have presented phenomenological theories of varying degrees of sophistication. Eddy viscosity models which are based on the simple Prandtl mixing length theory have also proven to be adequate in many meteorological and engineering studies [10–13]. Stability in the atmosphere due to thermal stratification can be conveniently expressed in terms of dimensionless numbers. For example, Obukhov [14] and Monin and Obukhov [15] employed a scaling length, L, which measures the shear forces relative to buoyant forces. The flux Richardson number, Ri_f, defined as the ratio of the buoyant suppression of eddy energy to the mechanical production of eddy energy, has also been used to represent the degree of stability in the atmosphere [6].

It is the purpose of the present work to investigate mass diffusion downstream of a steady infinite line source for both neutral and stably stratified conditions. The surface layer is modeled as a two-dimensional turbulent boundary layer. The governing partial differential equations are solved employing a two-layer eddy diffusivity model. For the stably stratified case, the above model is modified to account for the effects of stratification by introducing the flux Richardson number.

The present investigation serves to extend and improve a number of previous analytical studies. For example, Patankar and Taylor [16], based on Spalding's work [17], presented analytical solutions to the problem of mass diffusion from a line source in a neutral boundary layer. However, their results are only valid over a certain region of the flow field. Rao [18] and Rao *et al.* [19] studied mass diffusion in a neutral or thermally stratified turbulent shear layer. The work of both [18] and [19] is based on the Nee–Kovasznay theory [9], wherein a rate equation is postulated to govern the effective viscosity in an outer region of the boundary layer, while in the inner region the linear and logarithmic laws of the wall for the velocity profile were adopted. The linear law is based on a pure laminar sublayer adjacent to the wall, and the logarithmic law neglects the molecular contribution to diffusion. The concept of a laminar sublayer has been disproved as a result of measurement by Klebanoff [20] and Laufer [21]. The present study solves the governing partial differential equations without introducing the above simplifying assumptions.

ANALYSIS

Governing equations

The surface layer is modeled as a two-dimensional, turbulent boundary layer with zero pressure gradient. For normal wind velocities the governing partial differential equations are conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

conservation of momentum,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[(v+\varepsilon_M)\frac{\partial u}{\partial y}\right], \tag{2}$$

conservation of mass species,

$$u\frac{\partial m_1}{\partial x} + v\frac{\partial m_1}{\partial y} = \frac{\partial}{\partial y}\left[(v+\varepsilon_M)Sc_{\text{eff}}^{-1}\frac{\partial m_1}{\partial y}\right], \tag{3}$$

and potential temperature equation,

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y}\left[(v+\varepsilon_M)Pr_{\text{eff}}^{-1}\frac{\partial \theta}{\partial y}\right], \tag{4}$$

where the effective Schmidt and Prandtl numbers are defined by

$$Sc_{\text{eff}} = \frac{v+\varepsilon_M}{v+\varepsilon_D} = \frac{1+(\varepsilon_M/v)}{Sc^{-1}+Sc_t^{-1}(\varepsilon_M/v)}, \tag{5}$$

$$Pr_{\text{eff}} = \frac{v+\varepsilon_M}{v+\varepsilon_H} = \frac{1+(\varepsilon_M/v)}{Pr^{-1}+Pr_t^{-1}(\varepsilon_M/v)}, \tag{6}$$

where $Sc_t = \varepsilon_M/\varepsilon_D$ and $Pr_t = \varepsilon_M/\varepsilon_H$ are the turbulent Schmidt and Prandtl numbers, respectively. A model for the eddy viscosity for momentum, ε_M, which represents the turbulent contribution to the momentum transfer, is presented in the following paragraphs.

Turbulence model

The model of turbulence adopted in the present work is a two-layer model which involves a near-wall-region formulation and another formulation for the outer-wake-like region with continuity of ε_M imposed where the regions meet. In the outer part, the mixing length, l, is taken as uniform and proportional to the characteristic thickness of the boundary layer, y_l, while near the wall the Van Driest model [10] is employed. That is,

$$\varepsilon_M = K^2y^2\{1-\exp[-y\sqrt{(\rho\tau)}/\mu A^*]\}^2\frac{\partial u}{\partial y}, \quad \text{for} \quad 0 < yK < \lambda y_l \tag{7a}$$

$$\varepsilon_M = \lambda^2 y_l^2\frac{\partial u}{\partial y}, \quad \text{for} \quad \lambda y_l < yK. \tag{7b}$$

It is to be noted that the damping term in the expression (7a) is affected by the local shear stress, as suggested by Patankar and Spalding [11], rather than the wall shear stress proposed by Van Driest [10].

The above expressions, however, are only valid near the ground during neutral conditions, because turbulent diffusion is affected by the thermal stratification of the layer. Under neutral conditions, the potential temperature remains approximately constant with height and the vertical gradient of the static temperature is equal to the adiabatic lapse rate. Under inversion (stable stratification) conditions, the potential temperature increases with height and the negative buoyancy will suppress the turbulent diffusion. Therefore, the effect of the stratification has to be accounted for in the eddy viscosity model.

To include the effects of atmospheric stratification, Obukhov [14] suggested that the ratio (y/L) can be used as a stability parameter, where L is a scaling length given by

$$L = (\tau_w/\rho)^{3/2}/(kg/\theta_m)(-\dot{q}''/\rho c_p). \quad (8)$$

Under inversion conditions $\dot{q}'' < 0$ and $L > 0$, while for neutral conditions $\dot{q}'' = 0$ and $L \to \infty$. Monin and Obukhov [15] used the parameter (y/L) in their formulation of the dimensionless wind shear, ϕ, given by

$$\phi = \phi(y/L) = \frac{\partial u}{\partial y}\Big/[(\tau/\rho)^{1/2}/Ky]. \quad (9)$$

Under neutral conditions $y/L = 0$ and the quantity ϕ reduces to unity. They also suggested that ϕ takes the form

$$\phi = 1.0 + A(y/L), \quad (10)$$

where A is a constant. McVehil [22] found that for stable stratification the constant A has a value between 4.5 and 7.0.

Alternatively, the flux Richardson number, given by

$$Ri_f = g(-\dot{q}''/\rho c_p \theta_m)\Big/\left[(\tau_w/\rho)\frac{\partial u}{\partial y}\right] = (\tau_w/\rho)^{1/2}\Big/\left(LK\frac{\partial u}{\partial y}\right), \quad (11)$$

can be used as a stability parameter [6] and the dimensionless wind shear can assume a relation similar to equation (10), that is,

$$\phi = \phi(Ri_f) = 1 + A_1 Ri_f, \quad (12)$$

where A_1 is a numerical constant. A modified Prandtl mixing length l, which accounts for stratification, can be found from equation (9) and the following definition

$$(\tau/\rho) = l^2\left(\frac{\partial u}{\partial y}\right)^2 = \varepsilon_M \frac{\partial u}{\partial y} \quad (13)$$

to be

$$l = Ky/\phi. \quad (14)$$

The eddy viscosity model, adopted in the present work, is therefore written as follows:

$$\varepsilon_M = K^2y^2\{1-\exp[-y\sqrt{(\rho\tau)}/\mu A^*]\}^2\left(\frac{\partial u}{\partial y}\right)\Big/\phi^2 \quad \text{for} \quad 0 < yK < \lambda y_l \quad (15a)$$

$$\varepsilon_M = \lambda^2 y_l^2\left(\frac{\partial u}{\partial y}\right)\Big/\phi^2 \quad \text{for} \quad \lambda y_l < yK. \quad (15b)$$

For neutral conditions, ϕ has a value of unity, while for stably stratified conditions ϕ is represented by equation (12) with Ri_f given by equation (11). For neutral conditions the set of equations (1–3, 15) are solved. For the stably stratified case equation (4) is solved with the above set in order to compute the flux Richardson number.

Turbulent Schmidt and Prandtl numbers

The species and thermal energy equations can only be solved if one makes certain assumptions regarding the turbulent contributions to mass and heat transfer. Most investigators have solved the energy and/or species equation by assuming a constant value for Pr_t/Sc_t or they assumed Reynolds analogy would hold [11–13]. A review of the published experimental values of Pr_t (or Sc_t) [23, 24] demonstrates that Pr_t is not a constant but a function of distance from the wall. Wassel and Catton [13] presented a model for the variable Pr_t/Sc_t that compares well with experimental data. For a thermally stable layer the results of McVehil [22] and Webb [25] showed that $Pr_t = 1$ is a good approximation.

In the present work both constant and variable turbulent Schmidt number (or Prandtl number) are considered. A value of 0.9 and 1.0 for the constant Sc_t/Pr_t are used for the neutrally and stably stratified cases respectively. The variable Schmidt and/or Prandtl number is represented by

$$(Sc_t/Pr_t) = \frac{C_3}{C_1(Sc/Pr)} \frac{\left[1-\exp\left(-\frac{C_4}{(\varepsilon_M/\nu)}\right)\right]}{\left[1-\exp\left(-\frac{C_2}{(Sc/Pr)(\varepsilon_M/\nu)}\right)\right]} \quad (16)$$

where constants C_1, C_2, C_3, and C_4 are numerical constants.

Boundary conditions

Equations (1) and (2), conservation of mass and momentum, require specification of velocity components at the wall and at the free stream. They are given by

$$y = 0: \quad u = 0; \quad v = 0$$
$$y \to \infty: \quad u = u_e. \quad (17)$$

Equation (3), conservation of mass species, requires specification of the species concentration at the wall and in the free stream. They are

$$y = 0: \quad \mu Sc^{-1}\frac{\partial m_1}{\partial y} = 0 \quad \text{for} \quad x_s > 0$$
$$y \to \infty: \quad m_1 = m_{1,e}. \quad (18)$$

For thermally stratified environments, the potential temperature equation is solved to obtain the flux Richardson number. The temperature equation, equation (4), requires specification of the potential temperature at the wall and in the free stream. The boundary conditions are

$$y = 0: \quad \theta = \theta_w$$
$$y \to \infty: \quad \theta = \theta_e. \quad (19)$$

All boundary conditions, except the mass species concentration at the wall, are of the Dirichlet type. The species concentration at the wall, which is the maximum at a given streamwise location, can be extracted by satisfying the Newman boundary condition $\partial m_1/\partial y = 0$.

Method of solution

The numerical procedure employed here is a modification of the Patankar–Spalding procedure [11, 26] developed by Denny and Mills [27]. The method employs an implicit finite difference scheme to solve the parabolic partial differential equations by proceeding in a step-wise manner in the downstream direction. In order to transform equations (1)–(4) into a standard form suitable for solution by the finite difference method, let Ψ be the stream function defined by

$$\frac{d\Psi_w}{dx} = -\dot{m}''_w = -\rho v_w; \quad \frac{d\Psi_e}{dx} = -\dot{m}''_e = -\rho v_e, \tag{20}$$

and introduce the variable

$$\omega^2 = (\Psi - \Psi_w)/(\Psi_e - \Psi_w) = \int_0^y \rho u \, dy \bigg/ \int_0^{y_e} \rho u \, dy \tag{21}$$

into the governing equations and write them in the following general form:

$$\omega \frac{\partial \Phi}{\partial x} + (a' + b'\omega^2)\frac{\partial \Phi}{\partial \omega} = \frac{\partial}{\partial \omega}\left(c' \frac{\partial \Phi}{\partial \omega}\right) + \omega \, d'. \tag{22}$$

The symbol Φ stands for either of the dependent variables, a', b', and c' are variable coefficients, and d' is a source term. By obtaining the finite-difference equivalence of equation (22), the set of equations are solved by marching and successive substitution techniques. Initial profiles of the dependent variables are established at some upstream location. In forming the finite difference analogue of equation (22) an implicit scheme is adopted, thereby minimizing stability problems, to write

$$\Phi_i = A_i \Phi_{i+1} + B_i \Phi_{i-1} + C_i; \quad i = 2, 3, 4, \ldots, N, \tag{23}$$

which can be transformed into a simpler form [11]

$$\Phi_i = A'_i \Phi_{i+1} + B'_i \tag{24}$$

and solved by successive substitution.

RESULTS AND DISCUSSION

Results were obtained for a turbulent air boundary layer under both neutral and stably stratified atmospheric conditions. For purposes of comparison, flow conditions similar to those of past work [3, 18, 19, 28] were assumed and are shown in Fig. 1. To cover the range of parameters appropriate for comparison, numerical results were obtained along a flat plate for 24.4 m.

Thermophysical properties and empirical constants

The physical properties used for air are as follows:

$$c_p = 1017.4 \text{ J/kg °K}, \quad \rho = 1.20 \text{ kg/m}^3,$$
$$\mu = 1.786 \times 10^{-5} \text{ kg/m s}, \quad Sc = 0.72, \quad Pr = 0.72.$$

The empirical constants λ, K, and A^* used in the eddy viscosity model were taken to be 0.09, 0.435 and 26 respectively. These constants were evaluated by matching the computed solutions to reliable experimental data of velocity profiles and wall shear stress [11, 13, 26]. The adequacy of the model in predicting the flow field is discussed in detail in the literature, for example, [13]. Following [13], the empirical constants C_1, C_2, C_3, and C_4 of equation (16) were taken to be 0.21, 5.25, 0.2 and 5 respectively. The constant turbulent Schmidt number, Sc_t (or Pr_t), was taken to be 0.9 and 1.0 for the neutrally and stably stratified cases respectively [22, 25]. The constant A_1 in the expression for dimensionless wind shear, equation (12), was assumed to be 6.

Neutral atmospheric layer results

Figure 2 shows a plot of predicted ground level concentration and the parameter λ (defined as y at which $m_1 = \frac{1}{2} m_{1,\max}$) vs the distance downstream of a line source. Experimental results of Poreh and Cermak [3] and predictions of Rao *et al.* [19] are also plotted. Poreh and Cermak divided the region downstream of the source into four zones; the initial, intermediate, transition and final zones. Because the initial zone was very close to the source ($x_s < 0.914$ m) no reliable data was obtained there. They correlated the data in the intermediate zone ($x_s > 0.914$ m to x_s at which $\lambda/\delta \simeq 0.39$, where δ is boundary layer thickness) with

$$m_{1,\max} \rho u_e = 17.3 \, x_s^{-0.9} = 26.2 \, G x_s^{-0.9} \tag{25}$$

and

$$\lambda = 0.076 \, x_s^{0.8}. \tag{26}$$

Rao *et al.* [19] similarly correlated their predictions with

$$m_{1,\max} \rho u_e = 7.425 \, x_s^{-0.73} = 11.25 \, G x_s^{-0.73} \tag{27}$$

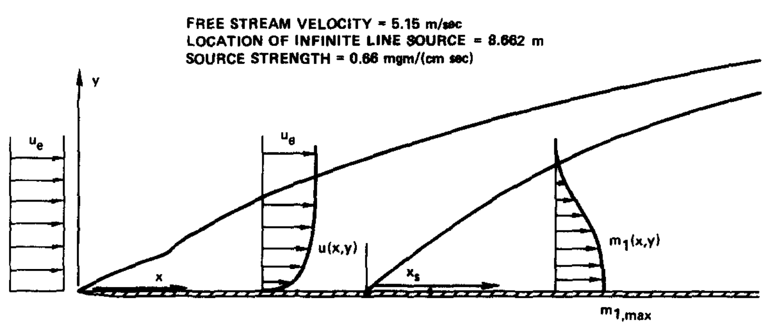

FIG. 1. Sketch of problem investigated.

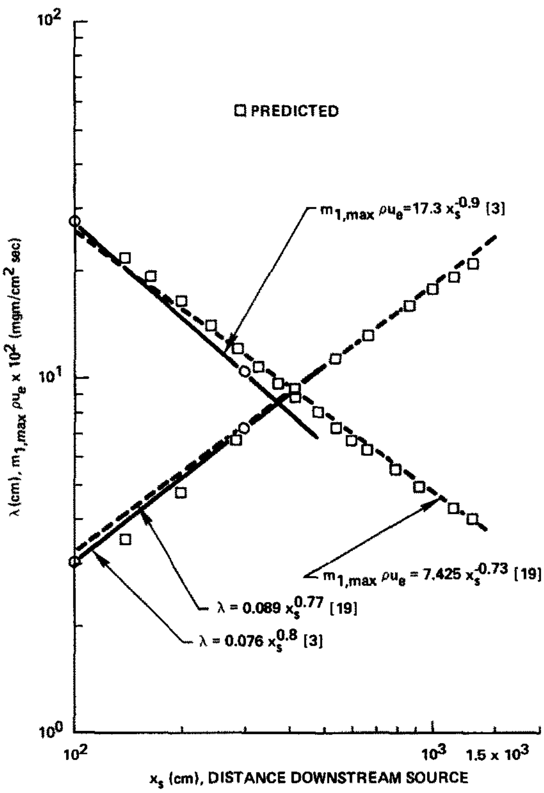

FIG. 2. Comparison of predicted $m_{1,\max}$ and λ with data of [3] and predictions of [19]. Neutral atmosphere $u_e = 5.15$ m/s, $G = 0.66$ mgm/cm s, $Sc_t = 0.9$.

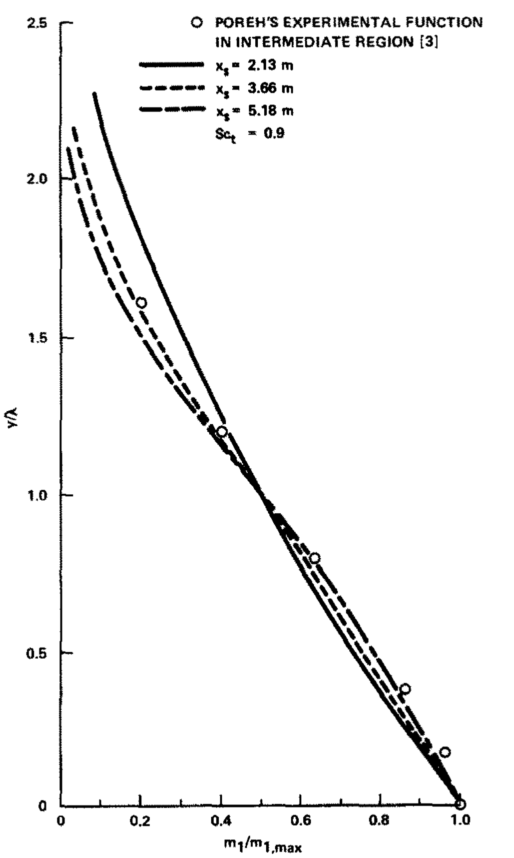

FIG. 3. Comparison of prediction with Poreh's fit to his experimental data. Neutral atmosphere.

and

$$\lambda = 0.089\, x_s^{0.77}. \tag{28}$$

It can be seen from Fig. 2 that the present results agree well with those of Rao *et al.* but have a maximum deviation of about 15%, in the values of $(m_{1,\max}\rho u_e)$, from the data of Poreh and Cermak at the end of the intermediate zone. Correlations of the present results will be presented later in the paper.

Figure 3 shows good agreement between predicted concentration profiles and Poreh and Cermak's similarity function $F(y/\lambda)$; $F(1) = 1/2$, in the intermediate zone. They assumed that such a function represents concentration profiles in the intermediate zone. Figure 4 also shows concentration profiles at different locations downstream of the source. As distance downstream increases, the ground concentration becomes more attenuated and the polutants diffuse more into the boundary layer, hence higher values of λ are obtained and the dimensionless concentration profile becomes flatter near the surface.

Figure 5 compares ground level concentration to those of Poreh and Cermak as well as Rao *et al.* in the final zone. Poreh and Cermak defined the final zone as the region where λ/δ remains constant and equals 0.64. Their measured ground concentration was approximated by

$$\rho m_{1,\max} = (G/0.55)/(u_e\,\delta). \tag{29}$$

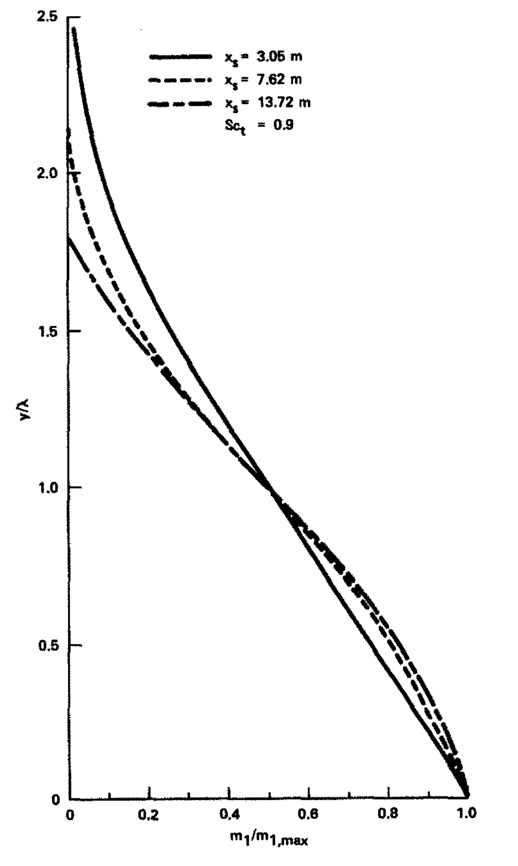

FIG. 4. Concentration profiles at different locations downstream. Neutral atmosphere.

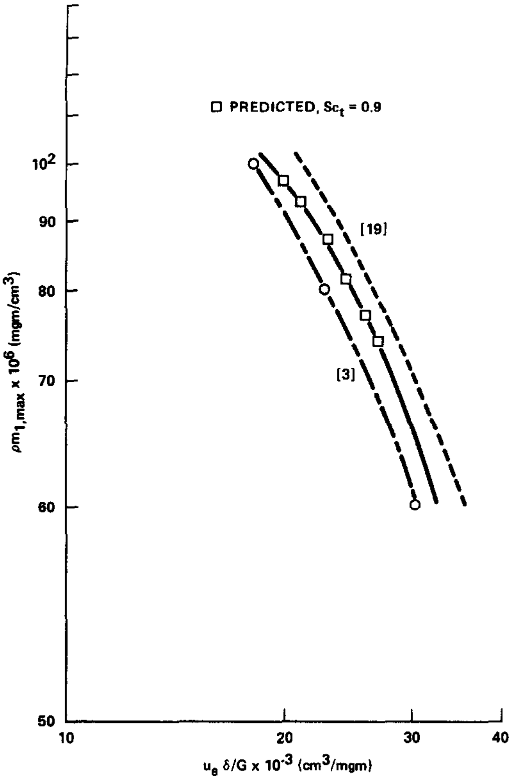

FIG. 5. Comparison of predicted ground level concentration with data of [3] (final zone) and results of [19]. Neutral atmosphere.

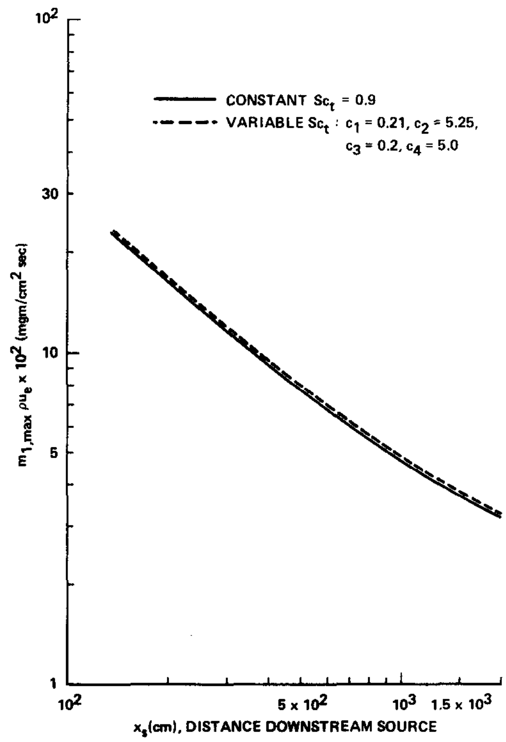

FIG. 6. Predicted ground level concentration for constant and variable Sc_t. Neutral atmosphere.

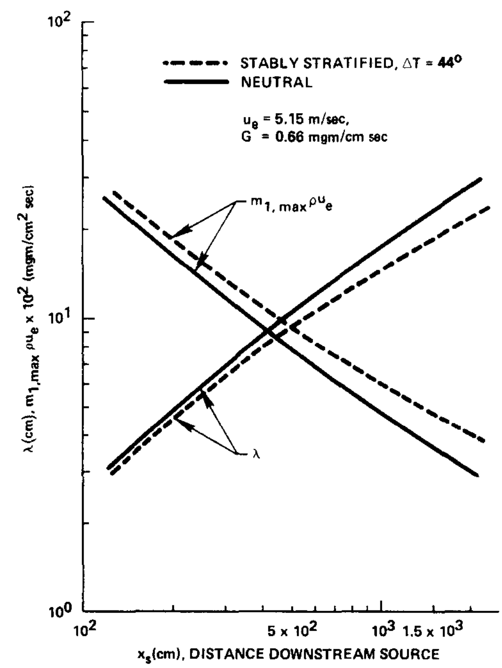

FIG. 7. Predicted ground level concentration and parameter λ for both stable and neutral atmospheres.

Similarly, Rao *et al.* correlated their results with

$$\rho m_{1,\max} = (G/0.47)/(u_e\,\delta_c) \qquad (30)$$

where $\delta_c = \lambda/0.64$, which reduces to

$$\rho m_{1,\max} = (G/0.47)/u_e\,\delta$$

if one assumes $\lambda/\delta = 0.64$. Figure 5 shows good agreement with both the experiments of Poreh and Cermak and the predictions of Rao *et al.* The maximum deviation is approximately 8%.

Figure 6 shows that the effect of the nonconstancy of Sc_t is very small. One can expect this result since near the wall the diffusion is mainly molecular. Also, since no mass species is crossing the wall the ground concentration is determined by satisfying the Newman boundary condition $\mu Sc_{\text{eff}}^{-1}(\partial m_1/\partial y) = 0$, i.e. $\partial m_1/\partial y = 0$, and the value of μSc_{eff}^{-1} very close to the wall has a minor effect on the results. Therefore, for the stably stratified case, only the results for the constant Sc_t (or Pr_t) will be presented.

Stably stratified atmospheric layer results

Results were obtained for stably stratified conditions for a temperature difference of 44 degrees and a temperature ratio of $\theta_w/\theta_e = 289°\text{K}/333°\text{K}$ across the boundary layer.

Figure 7 presents predicted values of ground level concentration and parameter λ as a function of distance downstream of the source for both neutral and stable atmospheres. It can be seen there that the ground concentration for the stable case is higher than that for the neutral one. This is due to thermal stratification, which reduces effective diffusivity as a result of the negative buoyancy. Hence, material diffuses away from the wall at a slower rate. One can,

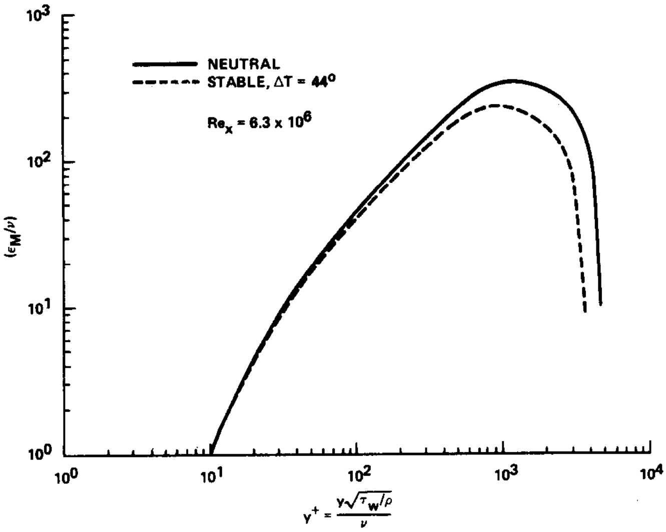

FIG. 8. Effect of thermal stratification on eddy viscosity.

therefore, expect that λ (the normal distance at which concentration is half the ground level) for a stable atmosphere would be less than the corresponding one for a neutral case. Figure 7 confirms this behavior. Adopting the local boundary layer heat flux, rather than the wall heat flux, in the definition of Ri was found to have a small effect on ground level species concentration.

The effect of thermal stratification on eddy viscosity, for Reynolds number 6.3×10^6, is shown in Fig. 8. It can be seen that thermal stratification suppresses the vertical turbulent mixing which is represented by ε_M/ν.

Predicted concentration profiles are presented in Fig. 9 and compared with Poreh's [28] and Malhotra and Cermak's [29] universal concentration distribution. They presented an empirical similarity curve,

$$m_1/m_{1,\max} = F(y/\lambda) = \exp\{-0.693(y/\lambda)^{1.8}\}, \quad (31)$$

for the concentration profiles in the intermediate zone.

Malhotra and Cermak [29] measured concentration profiles for neutral and unstable conditions and found that, for a line source the dimensionless species concentration can be represented by one universal dimensionless function, equation (31), and is independent of the stability condition. The dimensionless quantities involved, $m_{1,\max}$ and λ, however, were found to be strongly dependent on flow stratification.

In the present work it was assumed that the above universal function is applicable to a stably stratified flow. Figure 9 shows good agreement between predictions and expression (31). The dimensional quantities, $m_{1,\max}$ and λ strong dependence on stratification is depicted in Fig. 7 and Table 1.

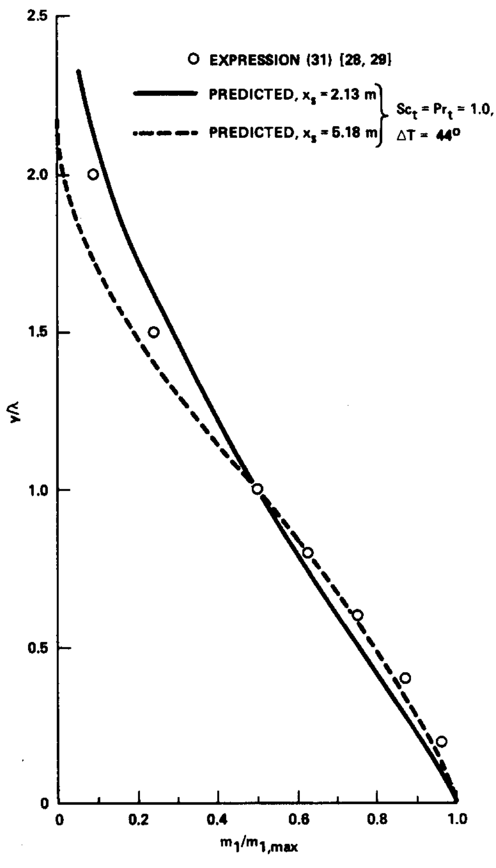

FIG. 9. Comparison of predicted concentration profiles with expression (31) [28, 29]. Stably stratified atmosphere.

Correlations of results

It can be seen from Fig. 7 that the ground level concentration, as well as the parameter λ, can easily be correlated as a function of distance downstream from the line source. Two regions can be identified which are for x_s less or greater than approximately 500 cm. This location corresponds to (λ/δ) of approximately 0.4. The following can be written

$$m_{1,\max}\,\rho u_e/G = a x_s^{-b} \quad (32)$$

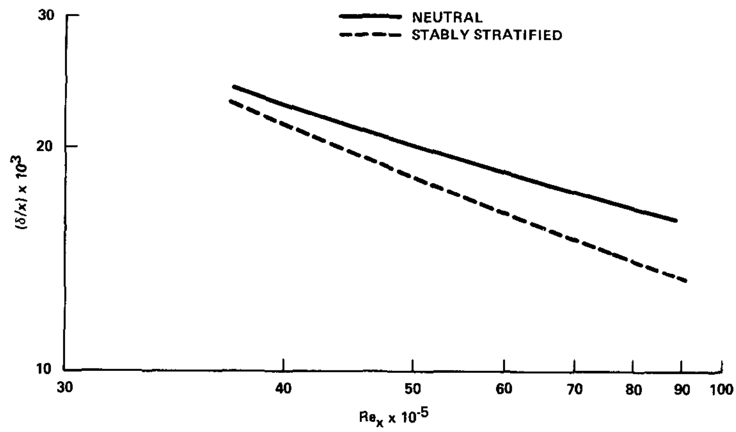

FIG. 10. Normalized boundary layer thickness vs local Reynolds number.

and

$$\lambda = cx_s^d \tag{33}$$

where the values of a, b, c, and d are given in Table 1. It

Table 1. Curve fit constants for equations (32) and (33)

		Neutral	Stable
$x_s < 500$	a	18.0473	15.0882
	b	0.8077	0.7520
	c	0.0450	0.0527
	d	0.8812	0.8354
$x_s > 500$	a	8.7212	6.2354
	b	0.6930	0.6121
	c	0.0775	0.1599
	d	0.7878	0.6518

should be noted that the above results correspond to the case where depletion of pollutants is not allowed, that is, the source strength G remains constant at any downstream station

$$G = \int_0^\infty \rho m_1 u \, dy = \text{constant}. \tag{34}$$

Finally, the scale of the problem can be seen from Fig. 10, which represents plots of the boundary layer thickness vs the local Reynolds number for both the neutral and stably stratified conditions.

SUMMARY AND CONCLUSIONS

Numerical solutions have been obtained for a turbulent boundary layer, under both the neutral and stably stratified conditions. Thermal stratification was represented by a flux Richardson number. Turbulent contribution to the mass and heat transfer was modeled by a two-layer eddy viscosity model. Neutral atmospheric layer results showed that the effect of variation of turbulent Schmidt number across the boundary layer on ground-level concentration was small; and the assumption of a constant value is adequate for engineering calculations. Results agreed well with experimental data and previous theoretical predictions. Execution computer time for a typical run was 20 and 25 s for neutral and stably stratified cases respectively, on an IBM 360/91.

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DIFFUSION A PARTIR D'UNE SOURCE LINEAIRE DANS UNE COUCHE ATMOSPHERIQUE STRATIFIEE NEUTRE OU STABLE

Résumé—On étudie la diffusion des polluants en aval d'une source linéaire attaquée transversalement par le vent dans une couche limite turbulente. Un modèle de turbulence selon Van Driest est utilisé avec une modification pour représenter la diffusion turbulente. L'effet de la stratification thermique sur la diffusion est représenté par un nombre de Richardson. Des solutions numériques des équations aux dérivées partielles sont obtenues par une méthode implicite aux différences finies. Des résultats sont présentés et comparés aux résultats expérimentaux et montrent un bon accord. On considère à la fois les nombres de Schmidt turbulents constants et variables.

DIE DIFFUSION VON EINER LINIENFÖRMIGEN QUELLE IN EINE NEUTRAL ODER STABIL GESCHICHTETE ATMOSPHÄRISCHE OBERFLÄCHENSCHICHT

Zusammenfassung—Es wird die Diffusion von Luftverunreinigungen stromabwärts einer quer zur Windrichtung angeordneten, linienförmigen Quelle in einer turbulenten Grenzschicht untersucht. Zur Darstellung der turbulenten Diffusion wird ein modifiziertes Van Driest-Turbulenzmodell verwendet. Der Einfluß der thermischen Schichtung auf die turbulente Diffusion wird durch eine Richardson–Zahl erfaßt. Die numerische Lösung der den Vorgang beschreibenden Grenzschichtgleichungen erfolgt über eine implizite finite Differenzenmethode. Die angegebenen Ergebnisse werden mit experimentellen Daten verglichen, wobei eine gute Übereinstimmung vorliegt. Es werden sowohl konstante wie veränderliche Schmidt–Zahlen berücksichtigt.

ДИФФУЗИЯ ОТ ЛИНЕЙНОГО ИСТОЧНИКА В НЕЙТРАЛЬНОМ ИЛИ УСТОЙЧИВО СТРАТИФИЦИРОВАННОМ ПОВЕРХНОСТНОМ СЛОЕ АТМОСФЕРЫ

Аннотация — На основе модифицированной модели Ван-Дриста исследуется диффузия загрязняющих веществ в турбулентном пограничном слое вниз по течению от расположенного поперечно ветру линейного источника. Влияние тепловой стратификации на турбулентную диффузию описывается числом Ричардсона для потока. С помощью неявного конечно-разностного метода получены численные решения дифференциальных уравнений пограничного слоя. Сравнение теоретических результатов с экспериментальными данными даёт хорошее соответствие. Рассмотрены как постоянные, так и переменные значения турбулентного числа Шмидта.